

STATISTIČKE METODE

FORMULE

$$\begin{aligned} \sum x_1^2 &= \sum X_1^2 - \frac{(\sum X_1)^2}{n} & \sum x_2^2 &= \sum X_2^2 - \frac{(\sum X_2)^2}{n} \\ \sum x_1 y &= \sum X_1 Y - \frac{(\sum X_1)(\sum Y)}{n} & \sum x_2 y &= \sum X_2 Y - \frac{(\sum X_2)(\sum Y)}{n} \\ \sum x_1 x_2 &= \sum X_1 X_2 - \frac{\sum X_1 \sum X_2}{n} & \sum y^2 &= \sum Y^2 - \frac{(\sum Y)^2}{n} \\ b_1 &= \frac{\sum x_2^2 \sum x_1 y - \sum x_1 x_2 \sum x_2 y}{\sum x_1^2 \sum x_2^2 - (\sum x_1 x_2)^2} & b_2 &= \frac{\sum x_1^2 \sum x_2 y - \sum x_1 x_2 \sum x_1 y}{\sum x_1^2 \sum x_2^2 - (\sum x_1 x_2)^2} \\ a &= \bar{Y} - b_1 \bar{X}_1 - b_2 \bar{X}_2 \\ C_{11} &= \frac{\sum x_2^2}{C} & C_{22} &= \frac{\sum x_1^2}{C} & C_{12} &= -\frac{\sum x_1 x_2}{C} & C &= \sum x_1^2 \sum x_2^2 - (\sum x_1 x_2)^2 \\ b_1 &= C_{11} \sum x_1 y + C_{12} \sum x_2 y & b_2 &= C_{21} \sum x_1 y + C_{22} \sum x_2 y \\ Q &= \sum Y^2 - \frac{(\sum Y)^2}{n} & Q_R &= b_1 \sum x_1 y + b_2 \sum x_2 y & Q_P &= \sum_{i=1}^n (Y_i - \hat{Y}_i)^2 = Q - Q_R \\ s_e &= \sqrt{\frac{\sum_{i=1}^n (Y_i - \hat{Y}_i)^2}{n-p-1}} & F &= \frac{\frac{Q_R}{p}}{\frac{Q_P}{n-p-1}} = \frac{s_R^2}{s_e^2} & t_1 &= \frac{b_1}{s_{b_1}} & t_2 &= \frac{b_2}{s_{b_2}} & t_i &= \frac{b_i}{s_{b_i}} \\ s_{b_1} &= \sqrt{s_e^2 C_{11}} & s_{b_2} &= \sqrt{s_e^2 C_{22}} & s_{b_i} &= \sqrt{s_e^2 C_{ii}} \\ b_i - t_{(n-p-1); \alpha} \cdot s_{b_i} &< \beta_i < b_i + t_{(n-p-1); \alpha} \cdot s_{b_i} \\ s_d &= \sqrt{s_e^2 (C_{11} - 2C_{12} + C_{22})} & t &= \frac{b_1 - b_2}{s_d} \\ s_a &= \sqrt{\frac{s_e^2}{n}} & \bar{Y} - t_{n-p-1; \alpha} \cdot s_a &< \mu < \bar{Y} + t_{n-p-1; \alpha} \cdot s_a \\ \hat{Y}_0 - t_{n-p-1; \alpha} \cdot s_{\hat{Y}_0} &< \mu_0 < \hat{Y}_0 + t_{n-p-1; \alpha} \cdot s_{\hat{Y}_0} \\ s_{\hat{Y}_0} &= \sqrt{s_e^2 \cdot \left[\frac{1}{n} + C_{11} \cdot (X_{10} - \bar{X}_1)^2 + C_{22} \cdot (X_{20} - \bar{X}_2)^2 + 2 \cdot C_{12} \cdot (X_{10} - \bar{X}_1)(X_{20} - \bar{X}_2) \right]} \\ \hat{Y}_0 - t_{n-p-1; \alpha} \cdot s_{Y'_0} &< Y_0 < \hat{Y}_0 + t_{n-p-1; \alpha} \cdot s_{Y'_0} & s_{Y'_0}^2 &= s_{\hat{Y}_0}^2 + s_e^2 \end{aligned}$$

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$$b'_i = \frac{b_i \cdot s_{X_i}}{s_Y} \quad s_{X_i} = \sqrt{\frac{\sum X_i^2}{n-1}}, i=1,2 \quad s_Y = \sqrt{\frac{\sum Y^2}{n-1}}$$

$$R^2 = \frac{Q_R}{Q} \quad R = \sqrt{R^2} \quad R^2 = b_1'^2 + b_2'^2 + 2 \cdot b_1' \cdot b_2' \cdot r_{23} \quad F = \frac{R^2 \cdot (n-p-1)}{(1-R^2) \cdot p}$$

$$R_A^2 = 1 - \frac{n-1}{n-p-1} \cdot \frac{Q_P}{Q} \quad R_A^2 = 1 - \frac{n-1}{n-p-1} \cdot (1-R^2)$$

$$R^2 = \frac{r_{12}^2 + r_{13}^2 - 2r_{12}r_{13}r_{23}}{1-r_{23}^2}$$

$$r_{X_1X_2} = r_{23} = \frac{\sum X_1X_2}{\sqrt{\sum X_1^2} \sqrt{\sum X_2^2}} \quad r_{X_1Y} = r_{12} = \frac{\sum X_1Y}{\sqrt{\sum X_1^2} \sqrt{\sum Y^2}} \quad r_{X_2Y} = r_{13} = \frac{\sum X_2Y}{\sqrt{\sum X_2^2} \sqrt{\sum Y^2}}$$

$$b_1 = b_{12.3} = \frac{s_1}{s_2} \cdot \frac{r_{12} - r_{13}r_{23}}{1-r_{23}^2} \quad b_2 = b_{13.2} = \frac{s_1}{s_3} \cdot \frac{r_{13} - r_{12}r_{23}}{1-r_{23}^2} \quad b_1' = \frac{r_{12} - r_{13}r_{23}}{1-r_{23}^2} \quad b_2' = \frac{r_{13} - r_{12}r_{23}}{1-r_{23}^2}$$

$$s_{b_1} = \sqrt{\frac{s_e^2}{\sum X_1^2 (1-r_{23}^2)}} \quad s_{b_2} = \sqrt{\frac{s_e^2}{\sum X_2^2 (1-r_{23}^2)}}$$

$$r_{12.3} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{1-r_{13}^2} \sqrt{1-r_{23}^2}} \quad r_{13.2} = \frac{r_{13} - r_{12}r_{23}}{\sqrt{1-r_{12}^2} \sqrt{1-r_{23}^2}}$$

$$t_1 = \frac{r_{12.3}}{\sqrt{1-r_{12.3}^2}} \sqrt{n-3} \quad t_2 = \frac{r_{13.2}}{\sqrt{1-r_{13.2}^2}} \sqrt{n-3}$$

$$R^2 = r_{12}^2 + r_{13.2}^2 (1-r_{12}^2) \quad R^2 = r_{13}^2 + r_{12.3}^2 (1-r_{13}^2)$$

$$F = \frac{Q_R - Q_R'}{s_e^2}$$

$$\begin{aligned} \sum Y &= a \cdot n + b \sum X + c \sum X^2, & \sum Y &= a \cdot n + c \sum X^2, \\ \sum XY &= a \sum X + b \sum X^2 + c \sum X^3, & \sum XY &= b \sum X^2, \\ \sum X^2Y &= a \sum X^2 + b \sum X^3 + c \sum X^4. & \sum X^2Y &= a \sum X^2 + c \sum X^4. \end{aligned}$$

$$Q = \sum y^2 = \sum Y^2 - \frac{(\sum Y)^2}{n}$$

$$Q_R = a \sum Y + b \sum XY + c \sum X^2Y - \frac{(\sum Y)^2}{n} \quad a = \bar{Y} - b\bar{X} - c(\bar{X})^2 \quad X_{\min(\max)} = -\frac{b}{2c}$$

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$$s_e^2 = \frac{\sum(Y - \hat{Y})^2}{n-3} \quad \rho = \sqrt{1 - \frac{\sum(Y - \hat{Y})^2}{\sum(Y - \bar{Y})^2}}$$

$$Y = \alpha\beta^X \varepsilon \quad \ln Y = \ln \alpha + X \cdot \ln \beta + \eta \quad Z = \ln Y, \quad A = \ln \alpha, \quad B = \ln \beta$$

$$\hat{Y} = ab^X \quad \hat{Z} = A + BX \quad A = \ln a \quad B = \ln b$$

$$B = \frac{\sum XZ - \frac{\sum X \sum Z}{n}}{\sum X^2 - \frac{(\sum X)^2}{n}} \quad A = \bar{Z} - B\bar{X} \quad B = \frac{\sum ZX}{\sum X^2} \quad A = \bar{Z} \quad p = (b-1) \cdot 100\%$$

$$Q_R = \frac{\left(\sum XZ - \frac{\sum X \sum Z}{n}\right)^2}{\sum X^2 - \frac{(\sum X)^2}{n}} \quad Q = \sum Z^2 - \frac{(\sum Z)^2}{n} \quad s_B = \sqrt{\frac{s_e^2}{\sum X^2 - \frac{(\sum X)^2}{n}}}$$

$$s_e = \sqrt{\frac{\sum_{i=1}^n (Z_i - \hat{Z}_i)^2}{n-2}} \quad s_e = \sqrt{\frac{Q_P}{n-2}}$$

$$Y = \alpha X^\beta \varepsilon \quad \ln Y = \ln \alpha + \beta \ln X + \eta \quad Z = \ln Y, \quad W = \ln X, \quad A = \ln \alpha$$

$$\hat{Y} = aX^b \quad \hat{Z} = A + bW \quad A = \ln a$$

$$b = \frac{\sum WZ - \frac{\sum W \sum Z}{n}}{\sum W^2 - \frac{(\sum W)^2}{n}} \quad A = \bar{Z} - b\bar{W} \quad Q_R = \frac{\left(\sum WZ - \frac{\sum W \sum Z}{n}\right)^2}{\sum W^2 - \frac{(\sum W)^2}{n}}$$

$$Y = \alpha + \beta \cdot \frac{1}{X} + \varepsilon \quad W = \frac{1}{X} \quad \hat{Y} = a + bW \quad b = \frac{\sum WY - \frac{\sum W \sum Y}{n}}{\sum W^2 - \frac{(\sum W)^2}{n}}$$

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$$\bar{Y}_{st} = \frac{\sum_{h=1}^L N_h \bar{Y}_h}{\sum_{h=1}^L N_h} \quad \bar{y}_{st} = \frac{\sum_{h=1}^L N_h \bar{y}_h}{\sum_{h=1}^L N_h} \quad \hat{Y}_h = N_h \bar{Y}_h \quad \hat{Y}_{st} = N \bar{Y}_{st}$$

$$\sigma_{\bar{y}_{st}}^2 = \sum_{h=1}^L \frac{N_h^2}{N^2} \frac{\sigma_h^2}{n_h} \quad \sigma_{\bar{y}_{st}}^2 = \sum_{h=1}^L \frac{N_h^2}{N^2} \frac{N_h - n_h}{N_h - 1} \frac{\sigma_h^2}{n_h}$$

$$s_{\bar{y}_{st}}^2 = \sum_{h=1}^L \frac{N_h^2}{N^2} \frac{s_h^2}{n_h} \quad s_{\bar{y}_{st}}^2 = \sum_{h=1}^L \frac{N_h^2}{N^2} \frac{N_h - n_h}{N_h} \frac{s_h^2}{n_h} \quad S_h^2 = \frac{\sum_{i=1}^{n_h} y_i^2 - \frac{\left(\sum_{i=1}^{n_h} y_i\right)^2}{n_h}}{n_h - 1}$$

$$\hat{Y}_h = N_h \bar{Y}_h$$

$$\bar{y}_{st,p} = \sum_{h=1}^L \frac{n_h}{n} \bar{y}_h \quad s_{\bar{y}_{st,p}}^2 = \frac{N-n}{N} \sum_{h=1}^L \frac{N_h}{N} \frac{s_h^2}{n} \quad s_{\bar{y}_{st,p}}^2 = \sum_{h=1}^L \frac{N_h}{N} \frac{s_h^2}{n} \quad s_{\bar{y}_{st,p}}^2 = \sum_{h=1}^L \frac{n_h}{n} \frac{s_h^2}{n}$$

$$p_{st} = \frac{\sum_{h=1}^L N_h p_h}{\sum_{h=1}^L N_h} \quad \hat{p}_{st} = \frac{\sum_{h=1}^L N_h \hat{p}_h}{\sum_{h=1}^L N_h} \quad \sigma_{\hat{p}_{st}}^2 = \sum_{h=1}^L \frac{N_h^2}{N^2} \frac{p_h q_h}{n_h} \quad \sigma_{\hat{p}_{st}}^2 = \sum_{h=1}^L \frac{N_h^2}{N^2} \frac{p_h q_h}{n_h} \frac{N_h - n_h}{N_h - 1}$$

$$s_{\hat{p}_{st}}^2 = \sum_{h=1}^L \frac{N_h^2}{N^2} \frac{\hat{p}_h \hat{q}_h}{n_h - 1} \quad s_{\hat{p}_{st}}^2 = \sum_{h=1}^L \frac{N_h^2}{N^2} \frac{\hat{p}_h \hat{q}_h}{n_h - 1} \frac{N_h - n_h}{N_h}$$

$$P(A) = \frac{m(A)}{n} \quad P(A) = \lim_{n \rightarrow \infty} \frac{f}{n}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(ABC)$$

$$P(A | B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) > 0$$

$$P(B) = \sum_{i=1}^n P(B | A_i) P(A_i), \quad (\forall i, j) A_i \cap A_j = \emptyset, \quad S = \bigcup_{i=1}^n A_i, \quad B \subset S$$

$$P(A_i | B) = \frac{P(B | A_i) \cdot P(A_i)}{\sum_{i=1}^n P(B | A_i) \cdot P(A_i)}$$

$$F(x) = P(X \leq x) = P(-\infty < X \leq x)$$

$$E(X) = \sum_i x_i p_i \quad V(X) = E(X^2) - [E(X)]^2$$

$$\text{cov}(X, Y) = E(XY) - E(X)E(Y)$$

$$V(X \pm Y) = V(X) \pm 2 \text{cov}(X, Y) + V(Y) \quad \text{cov}(X, Y) = \sum_i \sum_j x_i y_j p_{ij} - \sum_i x_i p_i \cdot \sum_j y_j p_j$$

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$$\hat{p} = \frac{\sum_{i=1}^k m_i}{k \cdot n} = \frac{\sum \hat{p}_i}{k} \quad s_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \quad GG = \hat{p} + 3 \cdot s_{\hat{p}}, \quad DG = \hat{p} - 3 \cdot s_{\hat{p}}.$$

$$\bar{c} \pm 3 \cdot \sqrt{\bar{c}}$$

$$\bar{X} = \frac{\sum_{i=1}^k \bar{X}_i}{k} \quad \bar{X} \pm 3 \cdot \frac{\bar{I}}{\sqrt{n} \cdot d_n} = \bar{X} \pm \bar{I} \cdot A$$

$$DG = B_1 \cdot \bar{I} \quad B_1 = 1 - 3 \cdot \frac{f_n}{d_n}$$

$$GG = B_2 \cdot \bar{I} \quad B_2 = 1 + 3 \cdot \frac{f_n}{d_n}$$

Табличне вредности за формирање контролне карте

n	d _n	f _n	A	B ₁	B ₂
4	2.06	0.880	0.729	0	2.28
5	2.33	0.864	0.577	0	2.11
6	2.53	0.848	0.483	0	2.00
7	2.70	0.833	0.419	0.076	1.92
8	2.85	0.820	0.373	0.136	1.86
9	2.97	0.808	0.337	0.184	1.82
10	3.08	0.797	0.308	0.223	1.78